

The η' propagator in quenched QCD*

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The calculation of the η' hairpin diagram is carried out in the modified quenched approximation (MQA) in which the lattice artifact which causes exceptional configurations is removed by shifting observed poles at $\kappa < \kappa_c$ in the quark propagators to the critical value of hopping parameter. By this method, the η' propagator can be accurately calculated even for very light quark mass. A determination of the topological susceptibility for quenched QCD is also obtained, using the fermionic method of Smit and Vink to calculate winding numbers.

1. INTRODUCTION

The properties of the flavor singlet pseudoscalar η' meson provide a unique phenomenological window on the topological structure of QCD. The mass of the η' is believed to arise from topologically nontrivial gauge configurations via the axial $U(1)_A$ anomaly. In a semiclassical treatment, the effect of instanton contributions to the $q\bar{q}$ annihilation (“hairpin”) diagram breaks the degeneracy between the η' and the flavor octet Goldstone bosons. In an effective chiral lagrangian description of QCD, the $q\bar{q}$ hairpin can be interpreted as an η' mass insertion. Within the large N_c approximation, Witten and Veneziano [1] showed that the topological theory of $U(1)_A$ breaking is consistent with the mass-insertion view, and that in this approximation, the η' mass in the chiral limit is proportional to the topological susceptibility χ_t of *pure gauge* (quenched) QCD.

$$m_0^2 = \frac{2N_f}{f_\pi^2} \chi_t \quad (1)$$

(Note: Here we take f_π normalized to have the experimental value of ≈ 96 MeV. This differs from that used in Ref. [2] by a factor $\sqrt{2}$.)

The study of the η' propagator in lattice QCD is of great interest, not only as a quantitative

check of the theory of the $U(1)$ problem, but also as a particularly sensitive probe of the differences between quenched and full QCD. For example, if the hairpin diagram behaves like a mass insertion, the quenched η' momentum-space propagator is expected to include a double Goldstone pole contribution.

$$\Delta_h(p^2) \propto \frac{1}{(p^2 + m_\pi^2)} m_0^2 \frac{1}{(p^2 + m_\pi^2)} \quad (2)$$

As a result, quenched chiral logs arising from virtual η' loops will complicate the chiral behavior of quenched QCD compared with that of the full theory[3]. One of the purposes of the study reported here is to investigate in detail the time-dependence of the hairpin contribution to the η' propagator and compare it with that expected from the double pole structure (2).

The method we use to calculate closed loops which originate at a given site of the lattice was introduced by Kuramashi, et al [4] in their original study of the η' mass. In this “allsource” technique, the quark propagator is calculated using a source given by a unit color-spin vector on *every* space-time point of the lattice. The closed quark loop from a given point is then calculated by assuming random-phase cancellation of other non-gauge-invariant terms.

A particular difficulty encountered in the η' hairpin calculation is the presence of rapidly increasing non-gaussian errors in the limit of small

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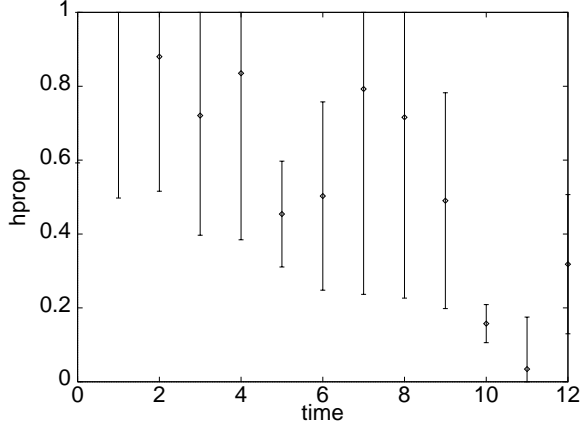


Figure 1. The unimproved hairpin propagator on a $12^3 \times 24$ lattice at $\beta = 5.7$ and $\kappa = .1675$

quark mass due to exceptional configurations. This problem is more severe in the hairpin calculation than it is in ordinary hadron spectrum calculations. Recently, the origin of the exceptional configuration problem has been traced to the presence of topological zero mode poles in the quark propagator which have been displaced to values of the hopping parameter below κ_c by lattice effects. [5] A practical method for removing this lattice artifact, the modified quenched approximation (MQA), has been proposed in Ref. [5]. An example of the MQA improvement of the hairpin propagator is shown in Figs. 1 and 2. The propagators shown in Figs. 1 and 2 were obtained on a $12^3 \times 24$ lattice at $\beta = 5.7$ with a hopping parameter of $\kappa = .1675$ ($m_q \approx 38$ MeV). The improvement of errors due to the MQA pole-shifting is impressive. For lighter quark masses, the data for the $12^3 \times 24$ lattice without the MQA improvement is unusable due to extremely large errors. After MQA improvement it is possible to accurately calculate the hairpin diagram down to very light quark masses. In our calculations, we have shifted all poles which were found below $\kappa = .1690$ and have calculated the hairpin for κ values up to .1685. It may be feasible to go to even lighter quark mass values, but the calculation of pole residues above $\kappa = .1690$ becomes rapidly more expensive in computer time.

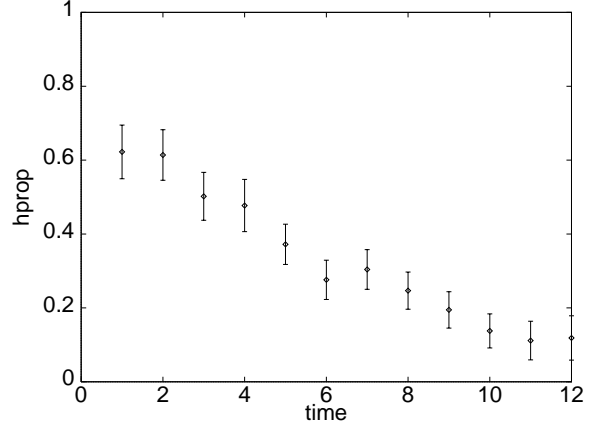


Figure 2. The MQA improved hairpin propagator on a $12^3 \times 24$ lattice at $\beta = 5.7$ and $\kappa = .1675$

2. THE η' PROPAGATOR

The hairpin diagram has been calculated on two ensembles of gauge configurations available in the ACPMAPS library. One ensemble included 200 configurations on a $12^3 \times 24$ lattice at $\beta = 5.7$. The other ensemble consists of 200 configurations at $\beta = 5.7$ on a $16^3 \times 32$ lattice. All together we have calculated the results for seven different values of hopping parameter ranging from .161 to .1685.

In general, the hairpin insertion may not be a simple p^2 -independent mass insertion, but instead may exhibit some p^2 -dependence. Expanding around the pion mass shell, we may write

$$m_0^2 \rightarrow h(p^2) = m_0^2 + \alpha(p^2 - m_{\pi^2}) + \dots \quad (3)$$

The second term corresponds to an addition to the η' kinetic term in the chiral lagrangian. The form of the hairpin insertion may be determined by studying the time-dependence of the hairpin propagator. If the hairpin vertex is a simple p^2 -independent mass insertion, the measured propagator at zero 3-momentum should behave according to a pure double-pole formula,

$$\tilde{\Delta}_h(\vec{p} = 0, t) = \frac{C}{4m_\pi^3} (1 + m_\pi t) \exp(-m_\pi t) + (t \leftrightarrow (T - t)) \quad (4)$$

The value of m_π is determined quite accurately

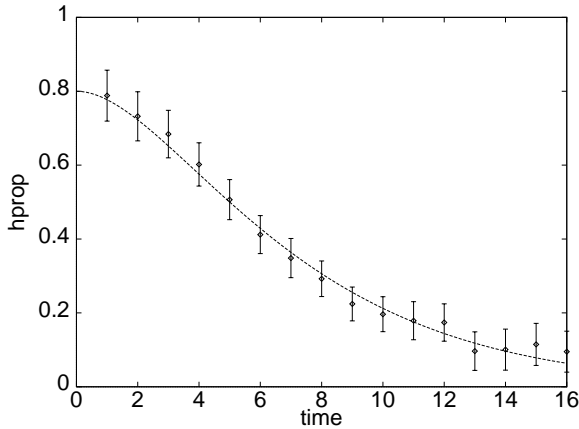


Figure 3. The MQA improved hairpin propagator on a $16^3 \times 32$ lattice at $\beta = 5.7$ and $\kappa = .1680$. The solid line is the time dependence of a pure double Goldstone pole, Eq. (4) expected if the η' hairpin vertex is a simple mass insertion.

from the valence pion propagator, so the only adjustable parameter in this fit is the overall normalization C . The second term in (3) would contribute a single-pole term to the propagator (2), which gives a term in (4) with pure exponential time dependence. Thus, both single and double pole terms are included by replacing the factor $C(1 + m_\pi T)$ in (4) by $(C_1 + C_2 m_\pi t)$. The preliminary results of this analysis indicate that there is very little p^2 dependence of the hairpin insertion. Indeed, the time dependence of the propagators is remarkably well described by the pure double-pole formula (4) over the entire observable range of time separations. An example for the $16^3 \times 32$ lattice at $\kappa = .1680$ is shown in Fig. 2.

By fitting the propagators to (4), and dividing out appropriate factors obtained from the valence pion propagator, we obtain a value for m_0 at each κ . The 16^3 results are consistent with those reported last year [2]. The finite volume increase observed in the 12^3 data [2] is found to be largely an effect of having more nearby visible poles on the smaller box. After the MQA shift, the results on the two box sizes are within a standard deviation of each other. The MQA analysis of f_π is not yet completed, but using previous results for

f_π , calculated on the $12^3 \times 24$ configurations, we obtain an effective chiral log parameter of $\delta \approx .04$ at $\kappa = .168$. The full results and comparison with other work will be presented elsewhere.

As a byproduct of the hairpin propagator calculation, we may calculate the integrated pseudoscalar charge density $Q_5 = \int \bar{\psi} \gamma_5 \psi d^4x$ on each lattice in the ensemble. As first suggested by Smit and Vink [6], this provides a fermionic method for determining the winding number ν of a gauge configuration, using the integrated anomaly equation, which gives $\nu = \lim_{m \rightarrow 0} m Q_5$. Using the allsource quark propagators, improved by the MQA pole shifting procedure, we have employed the Smit-Vink method to calculate the topological susceptibility $\chi_t = \langle \nu^2 \rangle / V$ (where V is the lattice 4-volume) for both the $12^3 \times 24$ and $16^3 \times 32$ ensembles. The results show only a mild κ dependence and can be sensibly extrapolated to zero quark mass. Using the scale $a^{-1} = 1.15$ GeV taken from charmonium, we obtain $\chi_t = 12.8 \pm 1.4 \times 10^{-4}$ GeV⁴ for the $12^3 \times 24$ lattice, and $\chi_t = 10.8 \pm 1.2 \times 10^{-4}$ GeV⁴ for the $16^3 \times 32$ lattice. This can be compared with 11.5×10^{-4} GeV⁴ from the WV formula (1).

REFERENCES

1. E. Witten, Nucl. Phys. **B156** (1979) 269; G. Veneziano, Nucl. Phys. **B159**, (1979) 213.
2. A. Duncan, E. Eichten, S. Perrucci, and H. Thacker, Nucl. Phys. B (Proc. Suppl.) **53** (1997) 256.
3. S. Sharpe, Phys. Rev. **D46** (1992) 3146; C. Bernard and M. Golterman, Phys. Rev. **D46**, (1992) 853.
4. Y. Kuramashi, M. Fukugita, H. Mino, M. Okawa, and A. Ukawa, Phys. Rev. Lett. **72** (1994) 3448.
5. See talk by E. Eichten in these proceedings and references cited therein.
6. J. Smit and J. Vink, Nucl. Phys. **B286** (1987) 485.